Eur. Phys. J. B **36**, 263–269 (2003)
DOI: 10.1140/epjb/e2003-00343-8

THE EUROPEAN PHYSICAL JOURNAL B

Landau-Zener interferometry for qubits

A.V. Shytov[1,2,a], D.A. Ivanov[3,b], and M.V. Feigel'man[1]

[1] L.D. Landau Institute for Theoretical Physics, Moscow 117940, Russia
[2] Institute for Theoretical Physics, University of California at Santa Barbara, USA
[3] Institute for Theoretical Physics, ETH-Zürich, 8093, Switzerland

Received 22 July 2003 / Received in final form 21 October 2003
Published online 8 December 2003 –

Abstract. One may probe coherence of a qubit by periodically sweeping its control parameter. The qubit is then excited by the Landau-Zener (LZ) mechanism. The interference between multiple LZ transitions leads to an oscillatory dependence of the energy absorption rate on the sweeping amplitude and on the period. This interference pattern allows to determine the decoherence time of the qubit. We introduce a simple phenomenological model describing this "interferometer", and find the form of the interference pattern.

PACS. 03.67.-a Quantum information – 85.25.Dq Superconducting quantum interference devices

During the last few years, a number of proposals for constructing quantum bits (qubits) from mesoscopic Josephson junctions have appeared [1–4] and first experimental results in this direction have been reported [5–11]. Large part of these qubits are actually different physical realizations of an externally controllable quantum double-well system, with nearly equal depths $E_{1,2}$ of both wells $|E_1 - E_2| \ll \omega_0 \ll E_1$ (here ω_0 is the oscillation frequency within a single well), and with the inter-well tunneling amplitude $\Delta \sim |E_1 - E_2|$. The above conditions ensure that higher eigenstates of the system are separated from the nearly degenerate doublet by a large gap (compared to Δ), and the probability of their excitation can be neglected. The energy difference $|E_1 - E_2|$ is controlled by an external time-dependent parameter $x(t)$, which is either voltage for the SET-based "charge" qubit [1,5], or magnetic flux through the Josephson junction loop for the "phase" qubit [3,4,6,8]. A review of recent results for both types of superconductive qubits can be found in [12]. Quantum manipulations with qubits involve varying in time both $x(t)$ and $\Delta(t)$ (as well as more complicated two-qubit manipulations). Since the overall scale of possible $E_{1,2}$ variations as function of control parameter X is very large compared to relevant values of Δ (e.g. it was more than 100 times larger in the design of reference [2]), fluctuations of $x(t)$ are expected to be one of the most important sources of dephasing in such qubits [4]. Indeed, the experimental data of reference [6] seem to confirm these expectations.

Thus the first problem to be addressed in the development of this type of qubits is to find a convenient probe which tests whether the device undergoes coherent evolution. Resonant absorption method was used in experiments of references [6,8], whereas Nakamura et al. have used time-domain manipulations [5]. It may be preferable to employ simpler methods to measure decoherence time of a qubit, without super-high-frequency (in the GHz range) manipulations. One such method (based on the measurement of static nonlinear Andreev conductance) was proposed in reference [13] for the specific case of superconducting phase qubit like those proposed in references [3,4]. Another low-frequency probe is the observation of Ramsey fringes, which was performed for superconducting qubits in [7,9]. In the present paper we propose and analyze a different method to determine the decoherence time of a qubit by a low-frequency non-resonant measurement.

The idea is to employ the Landau-Zener (LZ) non-adiabatic tunneling processes. If the control parameter $x(t)$ changes in such a way that the "collision region" with $\Delta \sim |E_1 - E_2|$ is traversed, the system may become excited from the lower to the upper level. If the parameter $x(t)$ is changed periodically, successive Landau-Zener tunneling events interfere, and such an interference allows to estimate the decoherence in the qubit. The interference picture is easiest to describe in the setup with the amplitude of the parameter sweep x_0 large compared to the width of the "transition region" (Fig. 1). If the dephasing time $t_{\rm deph}$ is larger than the period of the parameter sweep t_0, the interference between successive Landau-Zener transitions has an oscillatory dependence upon x_0 and t_0. The interference may be observed from

[a] e-mail: shytov@cmt.harvard.edu
[b] *Present address:* Paul Scherrer Institute, 5232 Villigen PSI, Switzerland

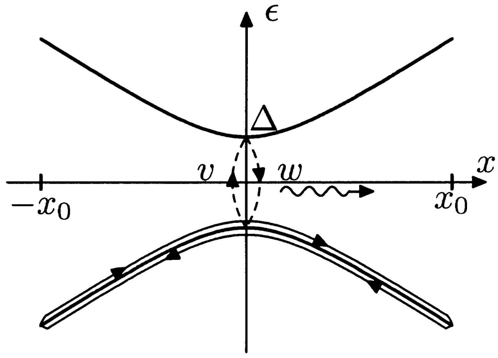

Fig. 1. Eigenvalues ϵ of the Hamiltonian (1) depend on the control parameter x. When the control parameter passes the Landau-Zener point (x=0), the qubit may be non-adiabatically excited with the probability v. The upper state may also decay into the lower one, dissipating the energy into the environment. The decay probability is w.

the rate of energy dissipation which is proportional to the occupation of the upper level. The precise form and the strength of the interference pattern is determined by the interplay of the coherent Landau-Zener transitions and of the spontaneous incoherent decay $E_2 \to E_1$. Performing interference experiments at a constant offset $x_{\rm off}$ in the oscillations of the parameter $x(t)$ and at different values of x_0 and t_0 allows to determine the rates of dephasing and of inelastic relaxation in the qubit.

Specifically, we model the qubit by the two-level Hamiltonian

$$H = \begin{pmatrix} x(t) + \hat{\xi}(t) & \Delta \\ \Delta & -x(t) - \hat{\xi}(t) \end{pmatrix}. \quad (1)$$

In this paper, we consider only one mode of qubit operation, namely varying the diagonal controlling parameter $x(t)$ while keeping the gap Δ fixed. We also assume that the main source of decoherence are thermal and quantum fluctuations of the control parameter around its intended value. Such fluctuations are described by the operator $\hat{\xi}(t)$. Phase fluctuations of Δ may also be incorporated into $\hat{\xi}(t)$ by an appropriate gauge transformation.

An example of the physical system leading to the Hamiltonian (1) is the superconducting phase qubit [2,6,13]. The control parameter $x(t)$ in this design is the magnetic flux through the qubit loop. The diagonal coupling to the environment is realized via coupling to the magnetic flux (including the fluctuations of the external electromagnetic field [4,19] and coupling to nuclear magnetic spins [18]). We also remark here that the diagonal coupling to the environment in the Hamiltonian (1) does not represent the most general form of coupling. In general, off-diagonal coupling may also be present, which would lead to the inelastic decay of the qubit states away from the Landau-Zener transition region. However, we assume that the off-diagonal coupling is already sufficiently suppressed: the qubit can preserve its occupation-number information in the "idle" regime (away from the Landau-Zener transition region), and the decoherence is determined by the diagonal coupling channel.

The quantum variable $\hat{\xi}(t)$ describes coupling to the collective degree of freedom of the external reservoir. It is a quantum variable corresponding to a collective degree of freedom of the reservoir. The usual model for the reservoir is an ensemble of harmonic oscillators [20]. Our treatment will be phenomenological and not involving the microscopic properties of the reservoir, therefore we do not explicitly include the reservoir in the Hamiltonian (1). However, we check the validity of our approach by comparing it to the microscopic calculation for the oscillator bath model (see Appendix).

Depending on the experimental conditions, the temperature of the reservoir may be either lower or higher than the gap Δ. We first consider the case of the reservoir temperature much smaller than the gap Δ, and later explain how the results are modified at higher temperature. Independently of the relation to the gap Δ, we assume that the reservoir temperature is always higher than the sweep frequency t_0^{-1}: this is necessary for our treatment of dephasing as a Gaussian noise and for our assumption of independent dephasing processes on different half-periods of the parameter sweep.

The effect of the coupling to $\hat{\xi}(t)$ is twofold. In the transition region ($x(t) \sim \Delta$), this coupling has non-vanishing matrix elements between the two adiabatic levels, and therefore leads to inelastic transitions between the levels. In the limit of the reservoir temperature much lower than Δ, the transitions occur mostly from the upper to the lower level, thus attenuating the transition probability [14]. Away from the transition region ($|x(t)| \gg \Delta$), the Hamiltonian (1) is almost diagonal, and the effect of $\hat{\xi}(t)$ is dephasing.

The control parameter $x(t)$ is swept periodically, with the amplitude x_0 and with the period t_0:

$$x(t) = x_0 \sin \frac{2\pi t}{t_0} + x_{\rm off} \ . \quad (2)$$

For simplicity, we first consider in detail the case of zero offset $x_{\rm off} = 0$, and then discuss the interference pattern at arbitrary $x_{\rm off}$.

Each time the control parameter passes the Landau-Zener point $x(t) = 0$, the Landau-Zener tunneling occurs. This tunneling is a quantum-mechanical process sensitive to the relative phase of the two states. Therefore, the energy absorption per period depends on the phase φ_n picked up far from LZ point. The latter phase is determined by both the sweep amplitude and the frequency (assuming $x_{\rm off} = 0$):

$$\varphi_n = 2 \int_0^{t_0/2} (x(t) + \xi(t))\, dt = \varphi + \delta\varphi_n \ , \quad \varphi = \frac{2x_0 t_0}{\pi}. \quad (3)$$

Here φ is the average phase picked up per half-period, and $\delta\varphi_n$ are its fluctuations. If we assume that the correlation time of $\hat{\xi}(t)$ is much shorter than t_0 (which is equivalent to assuming that the reservoir temperature is much higher than t_0^{-1}), the probability distributions of $\delta\varphi_n$ are Gaussian and uncorrelated on different half-periods (labeled by the integer n).

The amplitude x_0 is assumed to be much larger then the level-crossing region ($x_0 \gg \Delta$), and the period t_0 should be sufficiently large, so that the Landau-Zener transition probability [21] is small:

$$v = \exp\left(-\frac{\Delta^2 t_0}{2x_0}\right) \ll 1 \ . \tag{4}$$

Also, x_0 should not be too large so that the Hamiltonian (1) would still adequately describe the system. (For the superconducting phase qubit it implies that the amplitude of the flux modulation should be small compared to superconducting flux quantum.)

We describe the system evolution in terms of the two-level density matrix ρ. The evolution per one half-period of the parameter variation (2) is given by the master equation which includes the three effects: the coherent Landau-Zener transitions, the inelastic decay of the qubit, and the phase picked up during the system evolution away from the level-crossing region.

We separate the decoherence effect into the two parts: the dephasing away from the transition region and the inelastic relaxation in the transition region. This separation is possible if the transition region is narrow: $\Delta \ll x_0$. Instead of microscopically deriving the relevant couplings (see e.g. Refs. [14,15]), we include them phenomenologically as independent parameters in the master equation on the two-level density matrix. Both types of decoherence are assumed to be small. More precisely, we describe the decoherence by the two dimensionless parameters: the average phase fluctuation per one sweep, $u = \langle\delta\varphi_n^2\rangle$, and the probability of inelastic decay per one crossing of the transition region w. Our parameters w and u are proportional to the longitudinal and transverse relaxation rates, respectively (defined as $\Gamma_{\rm relax}$ and Γ_ϕ in Refs. [12,16]). These relaxation rates have contributions from different decoherence channels, and a priori there is no universal relation between them. Therefore instead of computing them microscopically, we introduce them phenomenologically as two independent parameters $u, w \ll 1$.

The three small parameters u, w, and v depend, in principle, on the period and amplitude of the oscillations of $x(t)$. We shall return to this dependence in the end of the paper.

The Landau-Zener transition, in the absence of inelastic events, is described by the unitary rotation of the density matrix:

$$\rho \mapsto S\rho S^\dagger, \qquad S = \begin{pmatrix} r & t \\ -t^* & r^* \end{pmatrix}, \tag{5}$$

where t and r are the amplitudes of the transition and of staying at the same adiabatic level, respectively. Amplitudes r and t satisfy unitarity condition $|r|^2 + |t|^2 = 1$. The magnitude of t determines the LZ transition probability (4): $|t|^2 = v$.

Let us assume first that the reservoir temperature is much lower than the gap Δ. Then the inelastic relaxation processes may be described as attenuation of the amplitude of being in the upper energy state (the case of higher reservoir temperatures will be discussed later, see equations (23) and (24) below):

$$\rho \mapsto V_w \rho V_w + w\rho^{(0)}\rho_{22} \ , \tag{6}$$

where

$$V_w = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-w} \end{pmatrix}, \qquad \rho^{(0)} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{7}$$

The equilibrium density matrix $\rho^{(0)}$ describes the system in the lower energy state. The quantity ρ_{22} is the diagonal element of the density matrix corresponding to the upper state. The first term in equation (6) describes the decay of the upper state. It attenuates the amplitudes of being in the upper energy state by $\sqrt{1-w}$. Thus, the probability that the upper state will not decay is $1-w$, and this allows to identify w as the inelastic decay probability. The second term in equation (6) describes the probability flow into the lower state due to the inelastic decay. This term is diagonal since the decay is assumed to be incoherent. Note that the equation (6) preserves the trace of the density matrix.

A complete solution of the dissipative dynamics requires simultaneously taking into account the Landau-Zener processes (5) and the dissipative processes (6) similarly to the treatment in references [14,15], which we do not attempt here. Instead, note that if the transition rates are small ($v, w \ll 1$), the elastic and inelastic processes may be considered independently. Thus, we simply combine equations (5, 6):

$$\rho \mapsto SV_w\rho V_w S^\dagger + w\rho_{22}\rho^{(0)} \ . \tag{8}$$

The matrices V_w and S do not commute, but the leading terms in w and v do not depend on the order of multiplication. We ordered S and V_w in equation (8) so that the trace of the density matrix is conserved.

Finally, the phase picked up far from the Landau-Zener point produces the relative phase rotation of the upper and lower states:

$$\rho \mapsto \Phi_n \rho \Phi_n^\dagger \ , \qquad \Phi_n = \exp\left(i\varphi_n\sigma_z/2\right), \tag{9}$$

where σ_z is the Pauli matrix.

In this way, the evolution of the density matrix per one sweep is described by the master equation

$$\rho \mapsto \Phi_n \left(SV_w\rho V_w S^\dagger + w\rho_{22}\rho^{(0)}\right)\Phi_n^\dagger. \tag{10}$$

We parameterize the density matrix as

$$\rho = \frac{1}{2}\left(a_0 + \mathbf{a}\sigma\right). \tag{11}$$

The scalar part $a_0 = 1$ remains constant, as required by the normalization of the density matrix. Then the dynamics is described by an equation for the polarization vector $\mathbf{a}$. The phase factor Φ_n in (9) rotates the vector $\mathbf{a}$ by the angle φ about z-axis. Similarly, the scattering matrix S in (5) rotates the vector $\mathbf{a}$ around some axis in xy-plane. One may redefine the phases of the upper and lower states so that S describes the rotation about x axis. (This transformation shifts all phases φ_n by a constant.) Thus the polarization vector $\mathbf{a}$ evolves per one sweep as

$$\mathbf{a}_{n+1} = Q_n \mathbf{a}_n + w\hat{\mathbf{z}} \ , \tag{12}$$

where Q_n is

$$Q_n = R_z(\delta\varphi_n) R_z(\varphi) R_x(\theta) A_w. \tag{13}$$

Here R_z and R_x are the rotation operators about z- and x-axes, and A_w describes the attenuation:

$$R_z(\varphi) = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix},$$

$$A_w = \begin{pmatrix} \sqrt{1-w} & 0 & 0 \\ 0 & \sqrt{1-w} & 0 \\ 0 & 0 & 1-w \end{pmatrix}. \tag{14}$$

The parameter θ is related to the Landau-Zener transition probability by $v = \sin^2\theta/2$. Note that the expression (13) is correct only to the leading orders in the small parameters w and v and should be treated as such.

Equation (12) must be solved for a stationary solution with fluctuating $\delta\varphi_n$. Since the phase fluctuations $\delta\varphi_n$ are assumed to be uncorrelated, averaging over these fluctuations amounts to averaging the evolution operator Q_n. After averaging $R_z(\delta\varphi_n)$

$$\overline{R_z(\delta\varphi_n)} = \begin{pmatrix} e^{-u/2} & 0 & 0 \\ 0 & e^{-u/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{15}$$

we arrive at the equation on the stationary solution $\bar{\mathbf{a}}$:

$$\bar{\mathbf{a}} = \bar{Q}\bar{\mathbf{a}} + w\hat{\mathbf{z}}, \quad \bar{Q} = \overline{R_z(\delta\varphi_n)} R_z(\varphi) R_x(\theta) A_w \ . \tag{16}$$

Solving (16) for $\bar{\mathbf{a}}$, we find the population of the upper level P_+:

$$P_+(\varphi) = \frac{1-a_z}{2} = \frac{v(u+w)}{\frac{1}{4}w(u+w)^2 + 2v(u+w) + 4w\sin^2\frac{\varphi}{2}}, \tag{17}$$

where we have kept only the leading terms in the small parameters w, u, and v.

This equation describes Lorentzian peaks positioned at $\varphi = 2\pi n$. The peaks are sharp if

$$w \gg uv, \tag{18}$$

in which case the width of the peaks $\delta\varphi$ is given by

$$(\delta\varphi)^2 = \frac{(u+w)^2}{4} + \frac{2v(u+w)}{w}. \tag{19}$$

In the case of an arbitrary non-zero offset $x_{\rm off}$ superimposed onto the periodic variation of the parameter (2), the two half-periods of the parameter sweep are no longer equivalent. The phase differences gained on odd and even half-periods differ by the corresponding phase offset $\varphi_{\rm off} = 2t_0 x_{\rm off}/\pi$: $\varphi_n = \varphi + \delta\varphi_n \pm \varphi_{\rm off}$, with the plus and minus sign for even/odd half-periods respectively. As a consequence, the period of the master equation (10) doubles, as it now includes two half-periods of the parameter sweep. In the interference pattern this produces secondary interference peaks at $\varphi = \pi + 2\pi n$. The relative intensities of the two peaks depend on the offset $\varphi_{\rm off}$, with the two intensities equal at $\varphi_{\rm off} = \pi/2 + \pi n$, and with one of the two peaks disappearing at $\varphi_{\rm off} = \pi n$.

A tedious but straightforward calculation results in the following extension of the formula (17) to the case of arbitrary $\varphi_{\rm off}$:

$$P_+(\varphi) = \frac{v(u+w)\left[1 + \cos\varphi\, \cos\varphi_{\rm off} + \frac{1}{8}(u+w)^2\right]}{D(\varphi, \varphi_{\rm off})}, \tag{20}$$

where

$$D(\varphi, \varphi_{\rm off}) = \frac{1}{2}w(u+w)^2 + 2w\sin^2\varphi + 2v(u+w)\left[1 + \cos\varphi\, \cos\varphi_{\rm off} + \frac{1}{8}(u+w)^2\right]. \tag{21}$$

This expression is again valid only to the leading orders in the small parameters u, v, and w (and coincides with Eq. (17) for $\varphi_{\rm off} = 0$ only in this limit). The terms $\frac{1}{8}(u+w)^2$ in the numerator and in the denominator are relevant only near the points $1 + \cos\varphi \cos\varphi_{\rm off} = 0$; away from these points, the terms $\frac{1}{8}(u+w)^2$ are beyond the precision of equation (20) and should be disregarded. Several examples of interference curves at different values of $\varphi_{\rm off}$ are plotted in Figure 2. Provided the condition of strong interference (18) is satisfied, the height of secondary peaks in (20) become equal to the background at

$$\varphi_{\rm off} \approx \frac{u+w}{2}. \tag{22}$$

Note that the secondary peak is much narrower than the primary one as long as its height is small: the width of small peaks is determined solely by the strength of decoherence processes $u + w$ (only the first term in Eq. (19)), whereas the width of the high primary peak involves the Landau-Zener amplitude v.

So far our discussion assumed the reservoir temperature $T_{\rm res}$ much lower than the gap Δ. Taking into account a finite reservoir temperature, the inelastic processes in (6) must include not only transition from the upper level to the lower one, but also the reverse transitions from the lower level to the upper one (absorbing energy from the

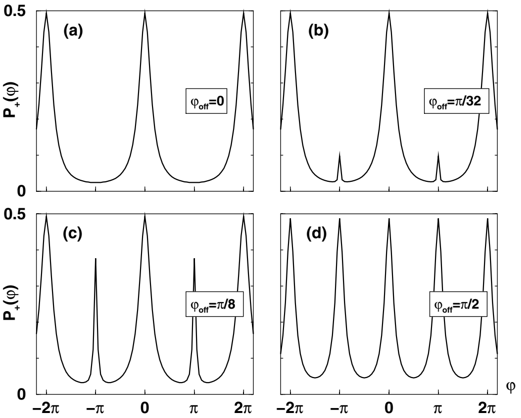

Fig. 2. Interference pattern $P_+(\varphi)$ as given by equations (20) and (21) for different values of the offset $\varphi_{\rm off}$. The non-adiabatic excitation probability $v = 10^{-3}$, the decay probability $w = 10^{-3}$, the dephasing factor $u = 0.1$. The four curves (a)–(d) correspond to $\varphi_{\rm off} = 0$, $\pi/32$, $\pi/8$, and $\pi/2$ respectively.

reservoir). The single transition probability w should then be replaced by the two probabilities w_1 and w_2. equation (6) is replaced by

$$\rho \mapsto V_w \rho V_w + w_1 \rho_1^{(0)} \rho_{22} + w_2 \rho_2^{(0)} \rho_{11} \ , \qquad (23)$$

where

$$V_w = \begin{pmatrix} \sqrt{1-w_2} & 0 \\ 0 & \sqrt{1-w_1} \end{pmatrix},$$
$$\rho_1^{(0)} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \rho_2^{(0)} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \qquad (24)$$

We give the expressions for the transition probabilities w_1 and w_2 in terms of the environment spectral function in the appendix. Repeating the same derivation as before, we arrive to the equation

$$\mathbf{a}_{n+1} = Q_n \mathbf{a}_n + (w_1 - w_2)\hat{\mathbf{z}} \qquad (25)$$

replacing equation (12), with Q_n given by the same expressions (13) and (14), except that now

$$A_w = \begin{pmatrix} \sqrt{(1-w_1)(1-w_2)} & 0 & 0 \\ 0 & \sqrt{(1-w_1)(1-w_2)} & 0 \\ 0 & 0 & 1-w_1-w_2 \end{pmatrix}. \qquad (26)$$

For small w_1 and w_2, this expression for A_w is equivalent to introducing the effective decay probability $w = w_1 + w_2$. Then the solutions may be obtained from our previous low-temperature results by a simple rescaling $\bar{\mathbf{a}} \mapsto \bar{\mathbf{a}}(w_1 - w_2)/(w_1 + w_2)$. In terms of the average occupation number of the upper level $P_+(\varphi)$, this translates to

$$P_+(\varphi) = P_+(\varphi, w = w_1 + w_2)\frac{w_1 - w_2}{w_1 + w_2} + \frac{w_2}{w_1 + w_2}, \qquad (27)$$

where $P_+(\varphi, w)$ in the right-hand side is given by equations (17) or (20). In other words, at finite reservoir temperatures, the interference pattern is simply rescaled by the factor $(w_1 - w_2)/(w_1 + w_2)$. At low reservoir temperatures, $T_{\rm res} \ll \Delta$, the ratio w_2/w_1 becomes exponentially small: $w_2/w_1 \sim \exp(-2\Delta/T_{\rm res})$. At high reservoir temperatures $T_{\rm res} \gg \Delta$, the probabilities w_1 and w_2 are close to each other, $(w_1 - w_2)/(w_1 + w_2) \sim \Delta/T_{\rm res}$, which accordingly decreases the amplitude (but not the sharpness) of the interference pattern $P_+(\varphi)$.

Experimentally, it may be possible to measure the energy absorption which is proportional to the population of the upper level P_+. By observing the appearance of the secondary peaks at varying $\varphi_{\rm off}$, it should be possible from (22) to determine the combined decoherence rate $u + w$. This is precisely the quantity which defines the quality of the qubit. The condition (18) should be fulfilled in order resolve well interference picture. Estimating $w \sim \Gamma_{\rm relax} t_0 \Delta / x_0$ and $u \sim \Gamma_\phi t_0$ (examples of estimates for longitudinal and transverse relaxation rates $\Gamma_{\rm relax}$ and Γ_ϕ for superconductive qubits can be found in [12,16]), and using (4), one finds the condition

$$\frac{\Gamma_{\rm relax}}{\Delta} \gg \frac{\Gamma_\phi x_0}{\Delta^2} \exp\left(-\frac{\Delta^2 t_0}{2x_0}\right). \qquad (28)$$

The condition (28) should be fulfilled together with inequalities $u, w \ll 1$. All these conditions together are compatible for low enough dephasing rate Γ_ϕ; taking for the sake of estimate $\Gamma_\phi/\Delta = 10^{-3}$ and $\Gamma_{\rm relax} \le \Gamma_\phi$ (cf. Ref. [16]), we find broad interval of allowed x_0 and t_0. Experimentally, values of $\Gamma_\phi/\Delta \sim 10^{-2}$ were measured in the first superconductive qubits [5,6], whereas much smaller normalized dephasing rate of order 10^{-4} was achieved for a non-quasiclassical device studied in reference [9].

It may be useful to perform measurements at different values of the amplitude x_0 and period t_0 of the parameter sweep. Both Landau-Zener transition probability v and the inelastic decay probability w depend only on the velocity at the transition point. If x_0 and t_0 are changed simultaneously so that x_0/t_0 is kept constant, v and w should also remain constant. At the same time, for short-range correlations of $\xi(t)$, the dephasing u scales linearly with t_0: $u = t_0 \Gamma_\phi$. Under these assumptions, from measurements at different x_0 and t_0 it may be possible to determine the dephasing rate Γ_ϕ, and the transition probabilities v, and w.

We are grateful to J.E.Mooij for a very useful discussion. This research was supported by NSF grant PHY99-07949, NWO-Russia collaboration grant, Swiss NF, RFBR grant # 01-02-17759, by Russian ministry of science, and by the program “Quantum Macrophysics” of Russian Academy of Science.

Appendix: The microscopic derivation of the equation for the density matrix

To establish connection between the microscopic Hamiltonian (1) and the phenomenological equations (6) and (23), we compute the density matrix directly. We consider the evolution of the qubit during one sweep, treating the coupling to the environment perturbatively, find correction to the density matrix $\hat{\rho}$, and compare it with the expansion of equation (23) in small w_1 and w_2. First, we rewrite the Hamiltonian (1) in the basis of adiabatic states:

$$\hat{H} = \hat{H}_0(t) + \hat{V}(t), \tag{29}$$

where the unperturbed Hamiltonian $\hat{H}_0(t)$ is diagonal,

$$\hat{H}_0 = \hat{\sigma}_z \epsilon_t; \qquad \epsilon_t = \sqrt{x^2(t) + \Delta^2}, \tag{30}$$

and the perturbation $\hat{V}(t)$ is given by

$$\hat{V}(t) = \hat{\xi}(t)(\cos\theta_t \hat{\sigma}_z + \sin\theta_t \hat{\sigma}_x); \quad \theta_t = \tan^{-1}\frac{\Delta}{x(t)}. \tag{31}$$

We neglect the probability of the LZ transition, and consider only the transitions due to the coupling to the environment $\hat{V}(t)$. In doing so, we assume that the characteristic energies involved are or order of $\epsilon_t \sim \Delta$, and the main t-dependence of the perturbation $\hat{V}(t)$ is due to fast fluctuation of $\hat{\xi}(t)$. Since θ_t changes essentially only on the large time scale $\Delta t_0/x_0$, we will treat it as a slow variable.

To compute the evolution of the density matrix $\hat{\rho}$ under the Hamiltonian (1), one may use Liouville equation in Heisenberg representation

$$\dot{\hat{\rho}}(t) = i\left[\hat{\xi}(t)\hat{U}(t), \hat{\rho}(t)\right], \tag{32}$$

where

$$\hat{U}(t) = \sin\theta_t e^{\frac{i}{2}\sigma_z\chi(t)}\sigma_x e^{-\frac{i}{2}\sigma_z\chi(t)}; \quad \chi(t) = 2\int_{-\infty}^{t} \epsilon_t\, dt, \tag{33}$$

where $\chi(t)$ is the phase difference between the two adiabatic states. The perturbation theory with respect to $V(t)$ gives, to the second order,

$$\hat{\rho}(t) = \hat{\rho}_0(t) + i\int_{-\infty}^{t} dt_1 \left[\hat{\xi}(t_1)\hat{U}(t_1), \hat{\rho}_0(t)\right] - \int_{-\infty}^{t} dt_1 \int_{-\infty}^{t_1} dt_2 \left[\hat{\xi}(t_1)\hat{U}(t_1), \left[\hat{\xi}(t_2)\hat{U}(t_2), \hat{\rho}_0\right]\right], \tag{34}$$

where $\hat{\rho}_0$ is the (time-independent) density matrix in zero approximation. Now, we average equation (34) over fluctuations of $\hat{\xi}(t)$. The first order term vanishes, and one has for $\delta\hat{\rho}(t) = \hat{\rho}(t) - \hat{\rho}_0$

$$\delta\hat{\rho}(t) = -\int_{-\infty}^{t} dt_1 \int_{-\infty}^{t_1} dt_2 \left[\hat{U}(t_1), \hat{U}(t_2)\, Q(t_1 - t_2)\hat{\rho}_0 - \hat{\rho}_0 \hat{U}(t_2)\, Q(t_2 - t_1)\right]. \tag{35}$$

Here $Q(t) = \langle\hat{\xi}(t)\hat{\xi}(0)\rangle$ is the correlation function of the environment. Note that since $\hat{\xi}(t)$ is quantum variable, $Q(t) \neq Q(-t)$. Rewriting the commutator in equation (35), one finds

$$\delta\rho_{11}(t) = -2\,\mathrm{Re}\int_{-\infty}^{t} dt_1 \int_{0}^{\infty} d\tau\, Q(\tau)\, \sin\theta_t \sin\theta_{t-\tau} \times \left(\rho_{11} e^{i\chi(t_1) - i\chi(t_1 - \tau)} - \rho_{22} e^{i\chi(t_1-\tau) - i\chi(t_1)}\right), \tag{36}$$

$$\delta\rho_{12}(t) = -\int_{-\infty}^{t} dt_1 \int_{0}^{\infty} d\tau [Q(\tau) + Q(-\tau)] \sin\theta_t \sin\theta_{t-\tau} \times \left(\rho_{12} e^{i\chi(t_1-\tau) - i\chi(t_1)} - \rho_{21} e^{i\chi(t_1) + i\chi(t_1-\tau)}\right), \tag{37}$$

and also $\delta\rho_{22}(t) = -\delta\rho_{11}(t)$, $\delta\rho_{21}(t) = \delta\rho_{12}^*(t)$. The dominant contribution to the integral over τ in equation (36) comes from the region $\tau \sim \epsilon_t^{-1}$, and one can use an approximation $\chi(t_1) - \chi(t_1 - \tau) \approx 2\tau\epsilon_{t_1}$. For the oscillator bath

$$\hat{H}_{\mathrm{env}} = \sum_i \omega_i a_i^+ a_i\ , \quad \hat{\xi}(t) = \sum_i \gamma_i (a_i e^{i\omega_i t} + a_i^+ e^{-i\omega_i t}) \tag{38}$$

integrals in equation (36) can be expressed in terms of the environment spectral function [20]

$$J(\Omega) = \sum_i \frac{\gamma_i^2}{\omega_i}\,\delta(\Omega - \omega_i) \tag{39}$$

as

$$\int_0^\infty Q(\tau) e^{i\omega\tau} d\tau = 2i\int_0^\infty \Omega J(\Omega) d\Omega \left[\frac{1 + n(\Omega)}{\omega + \Omega - i0} - \frac{n(\Omega)}{\omega - \Omega - i0}\right], \tag{40}$$

where $n(\Omega) = (\exp(\Omega/T_{\mathrm{res}}) - 1)^{-1}$ is Bose-Einstein distribution function, and T_{res} is the reservoir temperature.

After a straightforward calculation, one arrives to the correction to the density matrix at the end of one sweep:

$$\delta\rho_{11}(\infty) = -\rho_{11} w_2 + \rho_{22} w_1 \tag{41}$$

$$\delta\rho_{12}(\infty) = -\frac{1}{2}(w_1 + w_2)\rho_{12} + i\Phi\rho_{12}. \tag{42}$$

Here

$$w_1 = 4\pi\int_{-\infty}^{\infty} [1 + n(2\epsilon_t)]\, J(\epsilon_t) \sin^2\theta_t\, \epsilon_t\, dt \tag{43}$$

$$w_2 = 4\pi\int_{-\infty}^{\infty} n(2\epsilon_t) J(\epsilon_t) \sin^2\theta_t\, \epsilon_t\, dt \tag{44}$$

are the transition probabilities, and

$$\Phi = \int_{-\infty}^{\infty} \epsilon_t\, dt \sin^2\theta_t \int_0^\infty \frac{[2n(\Omega) + 1] J(\Omega)\Omega d\Omega}{\epsilon_t^2 - \Omega^2} \tag{45}$$

is the additional phase picked up during the sweep. This phase shift is due to the renormalization of the gap Δ due to the interaction between the qubit and the environment.

Comparison of equations (41) and (23) shows that the phenomenological equation (23) is correct in the perturbative limit. Also, since w_1 and w_2 contain $1+n(2\epsilon_t)$ and $n(2\epsilon_t)$ respectively, in the low-temperature limit ($T_{\rm res} \ll \Delta$) the decay rate w_1 is finite, while the excitation rate w_2 is thermally assisted: $w_2 \sim \exp(-2\Delta/T_{\rm res})$.

References

1. A. Shnirman, G. Schön, Z. Hermon, Phys. Rev. Lett. **79**, 2371 (1998); Yu. Makhlin, G. Schön, A. Shnirman, Nature **386**, 305 (1999)
2. J.E. Mooij et al., Science **285**, 1036 (1999)
3. L.B. Ioffe et al., Nature **398**, 679 (1999)
4. G. Blatter, L.B. Ioffe, V.B. Geshkenbein, Phys. Rev. B **63**, 174511 (2001)
5. Y. Nakamura, Yu. Pashkin, H.S. Tsai, Nature **398**, 786 (1999)
6. C. van der Wal et al., Science **290**, 773 (2000)
7. I. Chiorescu et al., Science **299**, 1869 (2003)
8. J.R. Friedman et al., Nature **406**, 43 (2000)
9. D. Vion et al., Science **296**, 886 (2002), and `cond-mat/0209315`
10. J. Martinis et al., Phys. Rev. Lett. **89**, 117901 (2002)
11. Y. Yu et al., Science **296**, 889 (2002)
12. Yu. Makhlin, G. Schön, A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001)
13. M.V. Feigel'man et al., J. Low Temp. Phys. **118**, 807 (2000)
14. Y. Gefen, E. Ben-Jacob, A.O. Caldeira, Phys. Rev. B **36**, 2770 (1987)
15. E. Shimshoni, Y. Gefen, Ann. Phys. **210**, 16 (1991)
16. A. Shnirman, Yu. Makhlin, G. Schön, `cond-mat/0202518`
17. T.P. Orlando et al., Phys. Rev. B **60**, 15398 (1999)
18. N. Prokofiev, P. Stamp, `cond-mat/0006084`
19. L. Tian et al., *"Decoherence of the superconducting persistent current qubit", in "Quantum mesoscopic phenomena and mesoscopic devices in microelectronics"*, edited by I.O. Kulik, R. Ellialtioglu (Kluwer Academic Publishers, 2000), `cond-mat/9910062`
20. A. J. Leggett et al., Rev. Mod. Phys. **59**, 1 (1987); U. Weiss, *Quantum dissipative systems*, 2nd edn. (World Scientific, Singapore, 1999)
21. L.D. Landau, Phys. Z. Sowjetunion **2**, 46 (1932); C. Zener, Proc. R. Soc. London, Ser. A **137**, 698 (1932)